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FEP Analysis and Markov Chains

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Abstract

Uncertainties related with underground CO₂ storage play a vital role in risk assessment with respect to Carbon storage and capture projects (CCS). The main purpose of risk assessment is to determine a qualitative and quantitative picture of hazardous processes or events. One makes a comprehensive inventory of risk factors, and stores the results in a FEP (Features, Events and Process) database. The properties of the geological system itself and natural or human-induced processes determine the future system properties.

The FEP's may interact. In this paper we propose to describe this interaction of FEPs within the framework of (discrete time) Markov Chains. In such an approach various *states* are defined. The system can “jump” from one state to another. The probabilistic evolution of the system can be followed, and conclusions can be drawn as to visit times of the various states. Also, the most likely ultimate fate of a system in dependence of initial state can be determined. This approach offers a complementary supporting tool for scenario-thinking, as it takes into account the evolution of all possible follow-ups of relevant physical processes and events quantitatively. It is not about just following a few scenario's. Without the machinery of Markov Chains this can hardly be done in full. An added bonus of this approach is that questions of policy makers, and of licensing authorities can be answered in a numerical way.

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1. Introduction

Assessing the feasibility of CO₂ storage systems entails safety and environmental issues. Their study relies on a methodological approach which encompass consequence analysis of (unwanted) CO₂ migration via faults, wells etc... towards the subsurface at a pre-determined site. Generally, such an approach starts with the hazard assessment focusing on establishing a comprehensive inventory of risk factors (Features, Events and Processes or FEPs) at the site at hand. Subsequently a selection of the most critical factors must be done, and they will lead to CO₂ leakage scenarios [1]. One then studies the scenario's in any concrete situation, i.e. for each proposed storage site. In this

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paper we will propose a way of “generalizing” scenario-thinking. We expect that with its help, it will become a little easier to answer (some of) the questions of policy makers, permit issuing authorities, and the general public.

2. Problem Definition

When TNO’s FEP database was first established in work by Wildenborg et al [1], it was intended to start establishing and evaluating the scenarios that might lead to hazardous situations. It contains static data but also addresses the linkage between the individual FEP’s and site-specific parameters. The database therefore is not only a suitable FEP selection tool, but also aims at assisting with scenario analysis. The work-flow incorporates the possibility of entering site specific data and FEP links. This should lead to a visual overview of FEP groups that might be important at any specific site, and ready to be used in a scenario analysis.

In practice, however, scenario’s seemed to be proposed by common sense, and the FEP database was typically used only as a kind of encyclopedia “in the background”. All kinds of visual aids to combine the information in the database linking the FEPS, turned out to be of little help. Working with the database as an assistant in scenario analysis in this way was somewhat uncomfortable. At the same time, it was felt that one should do more than use the database as a mere encyclopedia, or “shopping list” for crossing off items.

With this in mind, we propose a different outlook in which not scenario’s are the main players, but *states* of the system. The FEP database may be truly helpful in defining them. We will describe this outlook in the next chapter in broad outline.

3. Methodology

In our approach, we do not think in terms of scenario’s, but rather in terms of “states” of the subsurface.

Any possible combination of Events and Processes (EP’s) is a possible state. The initial, hopefully quiet, situation in the subsurface is also an event for our purposes. The “Features”-part from the FEP’s are properties of events and processes, and when they change that may well be an event in itself. In this sense they do play a role.

If we have N number of events and processes we have $(2^N - 1)$ possible subsets, i.e. states. For N = 10 that amounts to more than thousand states already. Considering the total number of events and processes in the database we would have an “unworkable” number of states. Therefore, from previous hazard risk assessment studies, we have chosen a limited number of states. How did we select them?

Below we present an example for explanatory purposes. It is a very simple example, that can be enlarged, and adapted for different situations. The future evolution of the geological storage system is going to be altered by events and processes. These alterations may take place in the CO₂ migration pathways (wells, seals, faults). Five general categories have been scrutinized.

1. Natural Changes in the subsurface (Seismicity)
2. Human activities in the subsurface (Drilling, Injection, Monitoring)
3. Unexpected Issues (Overpressure)
4. Geo-Chemical Phenomena (Dissolution, Chemical Weathering)
5. Geo-Mechanical Phenomena (Compaction, Uplift)

With help of the above scheme, we now define 10 states for a simplified description of the subsurface system.

State 1: Drilling phase

State 2: Injection phase

State 3: Monitoring in a “quiet” situation

- State 4: Monitoring : seal problem exists
- State 5: Monitoring : well problem exists
- State 6: Monitoring : seal & well problems exist
- State 7: Monitoring : seismic activity exists
- State 8: Monitoring : reservoir overpressure exists
- State 9: Upward flow through seal = first absorbing state (see below)
- State 10: Upward flow through well = second absorbing state

This set-up is very flexible. It can be made to include different subsurface characteristics and perceived hazardous situations. The site-specific FEP analysis by a team of specialists in any given situation will define the ultimate set of states.

For example, “seismic activity” may be defined by this team to possess a site-specific quantitative threshold. Or, the team wants to include an eleventh state: upward flow through seal and well simultaneously. Moreover, some states will not be included when assessed “a priori” as too improbable to merit such an inclusion.

Having defined the possible states of the system, we then will use the theory of Markov Chains to see what the likely evolution of such a system will be. Recall that in a Markov Chain we have a set of states $\{s_1, s_2, s_3, s_4, \dots, s_n\}$ with the property that the transition probability from one state to another does *not* depend on the history, but only on the present state of the system. In other words, the set of states are connected in the sense that a transition “ $i \rightarrow j$ ” is possible with a so-called transition probability denoted by $p(i, j)$, see [2]. The inclination for change is described with this transition probability, it is a probability per unit time that a jump from one specified state to another will occur.

It is up to the experts to produce viable transition probabilities in any site specific situation. We will not address here how that should be done. It is, nevertheless, a question of considerable practical importance, and there is a rich literature on the subject, tackling the psychology of elicitation as well as more formal issues [see for instance 3,4]. By establishing these numbers, we end up with the matrix of transition probabilities. We emphasize, again, that Markov theory pre-supposes that previous state-history must be deemed irrelevant.

Before working out the example we note the following. States 9 and 10 above are so-called absorbing states. They can be reached from the non-absorbing (transient) states (directly or via other transient states), but once reached the system stays there. We need to use the theory of *absorbing* Markov Chains for answering questions put forward below. The theory of these special chains is described in [2], whereas a general account of Markov Chains is in [5].

4. Working out the example

4.1. Questions to be asked

Let us ask ourselves which questions somebody in charge of site selection, or issuing permits, would ask. From a long-term perspective, then, the following questions are relevant.

- 1) If the system is at state j , what is the probability it ends up in each of the absorbing states in the chain?
- 2) If in state j , how much time will the system reside on average in any of the transient states before going into an absorbing state. And, consequently, how much time will elapse before absorption?

The relevance of the first question is manifest if there are several absorbing states are feasible, but with different potential risks. In our example gas may diffuse out of a leaky reservoir in a final state, or force its way up through a leaky well. One possibility may be deemed more fearsome than the other. You, as a policy maker, want to know what the odds are for the final situation.

It is important to note here that in an absorbing chain the system will eventually end up in one of the absorbing states. This is a mathematical fact. The really important question is what timescales are involved. Why is that? During the evolution of the system it will visit several states, potentially, before being caught in an absorbing state. These visits may require financial efforts from humans in terms of mitigation activities, or they are just disliked by themselves. Policy makers will want to know how long the system is expected to reside in such “expensive” states. Questions like this are linked with public acceptance and political sensitiveness, &. The relevant timescales are the ones answered in the second question above.

4.2. The absorbing chain

The following transition probabilities in the above example are set by the authors. The non-zero transition probabilities are:

	State 1	State 2	State 3	State 4	State 5	State 6	State 7	State 8	State 9	State 10
State 1	0.5	0.5								
State 2		0.99	8.0E-03					2.0E-03		
State 3			0.9939	1.0E-03	4.0E-03	1.0E-04	1.0E-03			
State 4				0.9097		8.0E-02	3.0E-04		1.0E-02	
State 5			0.4		0.595	1.0E-03	1.0E-03			3.0E-03
State 6				0.4		0.586	1.0E-03		1.0E-02	3.0E-03
State 7			0.947	1.0E-02	4.0E-02	3.0E-03				
State 8				1.0E-03				0.999		
State 9									1.0	
State 10										1.0

Table 1: Matrix showing the probabilities for changing states.

	State 1	State 2	State 3	State 4	State 5	State 6	State 7	State 8
State 1	2.00	100.00	726.90	77.98	7.26	15.27	0.77	200.00
State 2	0	100.00	726.90	77.98	7.26	15.27	0.77	200.00
State 3	0	0	900.20	77.51	8.98	15.22	0.95	0
State 4	0	0	33.70	79.83	0.34	15.44	0.07	0
State 5	0	0	891.37	76.94	11.37	15.12	0.94	0
State 6	0	0	34.71	77.32	0.35	17.37	0.08	0
State 7	0	0	888.59	77.51	8.97	15.23	1.94	0
State 8	0	0	33.70	79.83	0.34	15.44	0.07	1000.00

Table 2: Fundamental Matrix N. Matrix shows calculated time (in years) for changing states.

The “fundamental matrix” $N(i,j)$ represents the expected residence time in transient state j when the system started in transient state i . This matrix is 8×8 in our case, since we have 8 transient states. This fundamental matrix is basic for understanding the durations involved. It converts the rather abstract transition probabilities into a more comfortable entity: the time you expect the system to spend in the various transient states before reaching one of the absorbing states. Assuming the unit of transition probabilities is yr^{-1} , the computed durations in N are in years.

	State 9	State 10
State 1	0.932	0.068
State 2	0.932	0.068
State 3	0.927	0.073
State 4	0.953	0.047
State 5	0.921	0.079
State 6	0.947	0.053
State 7	0.927	0.073
State 8	0.953	0.047

Table 3 : Absorption Matrix B

The “absorption -matrix” $B(i,j)$ in table 3 represents the probability to end up in absorbing state j , once you started in transient state i . This is a 8×2 matrix in our case, because of our 8 transient states and 2 absorbing states.

The time to absorption from each initial transient state is the expected time to absorption from each of these transient states. Its values can be trivially derived from matrix N by appropriate summation.

When inspecting the results you will notice there is something striking with one of the states. In state 8 there are just two transitions: the transition to an absorbing state, and staying as is. One might envisage the possibility of mitigation actions, consequent upon discovering a problem. This, then, might lead to inclusion of a “mitigation” state. We will not pursue this matter in detail. In general, “experimenting” with systems like this will sharpen the insights as to the necessary details for describing the evolution.

5. Conclusions

In this paper, we dealt with CO_2 storage systems. We advocate the usage of “states” in order to compute relevant evolutionary timescales in such a system. We thereby draw upon the theory of absorbing Markov Chains. “State-thinking” enables the generalization of “scenario-thinking” as it takes care of all kinds of transitions, not only the major ones generally considered in a scenario. Furthermore, by numerically “experimenting” with state dynamics, one may acquire a more acute awareness for what information is crucial in answering questions for policy makers.

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